

Alloy yield strength modeling with MatCalc (rel. 5.61.0057)

P. Warczok





Model overview

- Contributions to yield strength, σ_{YS}
 - Intrinsic strength, σ_i
 - Work hardening, σ_{disl}
 - Grain/subgrain boundary strengthening, σ_{gb} , σ_{sgb}
 - Solid solution strengthening, σ_{ss}
 - Precipitation strenghtening, $\sigma_{\!prec}$

$$\boldsymbol{\sigma}_{YS} = f(\boldsymbol{\sigma}_{i}, \boldsymbol{\sigma}_{disl}, \boldsymbol{\sigma}_{gb}, \boldsymbol{\sigma}_{sgb}, \boldsymbol{\sigma}_{ss}, \boldsymbol{\sigma}_{prec})$$



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$$\boldsymbol{\sigma}_{YS} = f(\boldsymbol{\sigma}_{i}, \boldsymbol{\sigma}_{disl}, \boldsymbol{\sigma}_{gb}, \boldsymbol{\sigma}_{sgb}, \boldsymbol{\sigma}_{ss}, \boldsymbol{\sigma}_{prec})$$



Intrinsic strength, σ_i

•	Precipitation domains ? ×
Precipitation domains nickelmatrix New Remove Rename	General Mech. Props MS Evolution Solute trapping Special General Solid Solution Precipitation Mechanical properties Young's Modulus [Pa] 208e9 Taylor factor (2.5-3.1) 2,6 Poisson's ratio Matrix strength evaluation Basic strength [Pa] 21,8e6 Hall-Petch coeff (gb/sgb) 0,16e6 / Disl. strengt. coeff. (a1/a2) 0.5 / 0.3 Total strength coupling coefficients Coeff. thermal + athermal (1.8) 1.8 Strain rate sensitivity (temporary) exp_m 0.05
	Cancel OK

variables		5	×
variables	value		^
kinetics: pd strength TYSB\$*			
TYSB\$nickelmatrix	2.18e+07		۷
category: kinetics: pd strength expression: TYSB\$* legal unit qualifiers: *none* -> basic yield strength of precipitation domain			



Model overview

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 - Precipitation strenghtening, σ_{prec}

$$\boldsymbol{\sigma}_{YS} = f(\boldsymbol{\sigma}_{i}, \boldsymbol{\sigma}_{disl}, \boldsymbol{\sigma}_{gb}, \boldsymbol{\sigma}_{sgb}, \boldsymbol{\sigma}_{ss}, \boldsymbol{\sigma}_{prec})$$



• Taylor equation

$$\sigma_{disl} = A \alpha G b \sqrt{\rho}$$

- lpha Taylor factor
- G Shear stress
- b Burger's vector
- ho Dislocation density



- Taylor equation
 - $\sigma_{disl} = A \alpha G b \sqrt{\rho}$
 - lpha Taylor factor
 - G Shear stress
 - b Burger's vector
 - $ho\,$ Dislocation density



	Precipitation domains	? ×
Precipitation domains	General Mech. Props MS Evolution Solute trapping Special General Solid Solution Precipitation Mechanical properties Young's Modulus [Pa] 208e9 Taylor factor (2.5-3.1) 2,6 Poisson's ratio ,33	
	Matrix strength evaluation Basic strength [Pa] 21,8e6 Hall-Petch coeff (gb/sgb) 0,16e6 / 0.0e6 Disl. strengt. coeff. (a1/a2) 0.5 / 0.3 Total strength coupling coefficients	
New Remove	Coeff. thermal + athermal (1.8) 1.8 Strain rate sensitivity (temporary) eps_dot_ref 1.0 exp_m	
Rename	Cancel	ОК



17

Taylor equation

 $\sigma_{disl} = A \alpha G b \sqrt{\rho}$

- lpha Taylor factor
- G Shear stress
- b Burger's vector
- $ho\,$ Dislocation density

		Precipitation	n domains		? ×
Precipitation domains	General	Mech. Props	MS Evolution	Solute trapping Special]
nickelmatrix	FCC_A1	ynamic matrix pha:	se		
	-Microstru equilibriu	cture parameters m dislocation densi	 ity [m-2]	1.0e11	
	initial gra	in diameter [m] ograin diameter [m]	20e-6	elongation factor 1	
	Burger's	vector mar	nual value [m] 2	2.5e-10	
New Remove					
				Cance	ОК



•

- Taylor equation
 - Two parameter model

$$\sigma_{disl} = A_1 \alpha G b \sqrt{\rho_1} + A_2 \alpha G b \sqrt{\rho_2}$$

	Precipitation domains	?	x
Precipitation domains	General Mech. Props MS Evolution Solute trapping Special		
New Remove Rename	General Solid Solution Precipitation Mechanical properties Young's Modulus [Pa] 208e9 Taylor factor (2.5-3.1) 2,6 Poisson's ratio ,33 Matrix strength evaluation Basic strength [Pa] 21,8e6 Hall-Petch coeff (gb/sgb) 0,16e6 / 0.0e6 Disl. strengt. coeff. (a1/a2) 0.5 / 0.3 Total strength coupling coefficients Coeff. thermal + athermal (1.8) 1.8 Strain rate sensitivity (temporary) exp_m 0.05		
	Cancel	OK	



- Taylor equation
 - Two parameter model

 $\sigma_{disl} = A_1 \alpha G b \sqrt{\rho_1} + A_2 \alpha G b \sqrt{\rho_2}$

variables	value	^
kinetics: pd strength TDS\$*		
TDS\$nickelmatrix	7.1404e+06	¥
category: kinetics: pd strength expression: TDS\$nickelmatrix legal unit qualifiers: *none* -> dislocation yield strength contribution in precipitation domain		



Model overview

- Contributions to yield strength, σ_{YS}
 - Intrinsic strength, σ_i
 - Work hardening, σ_{disl}

• Grain/subgrain boundary strengthening, $\sigma_{\!gb}$, $\sigma_{\!sgb}$

- Solid solution strengthening, σ_{ss}
- Precipitation strenghtening, σ_{prec}

$$\boldsymbol{\sigma}_{YS} = f(\boldsymbol{\sigma}_{i}, \boldsymbol{\sigma}_{disl}, \boldsymbol{\sigma}_{gb}, \boldsymbol{\sigma}_{sgb}, \boldsymbol{\sigma}_{ss}, \boldsymbol{\sigma}_{prec})$$



• Hall-Petch equation

 $=\frac{k_{gb}}{\Gamma}$ k_{sgb} $\sigma_{_{gb}}$ σ_{sgb} =

- D Grain diameter
- δ Subgrain diameter
- k_n Constant



Ksgb

 σ_{sgb}

• Hall-Petch equation

2_4444

gb

- D Grain diameter
- δ Subgrain diameter
- k_n Constant

• • • • • • • • • • • • • • • • • • •	Precipitation domains	?	x
Precipitation domains	General Mech. Props MS Evolution Solute trapping Special		
nickelmatrix New Remove	General Mich. Props Mis Evolution Solute trapping Special General Solid Solution Precipitation Mechanical properties Young's Modulus [Pa] 208e9 Taylor factor (2.5-3.1) 2,6 Poisson's ratio ,33 Matrix strength evaluation Basic strength [Pa] 21,8e6 Hall-Petch coeff (gb/sgb) 0,16e6 / 0.0e6 Disl. strengt. coeff. (a1/a2) 0.5 / 0.3 Total strength coupling coefficients Coeff. thermal + athermal (1.8) 1.8 Strain rate sensitivity (temporary) exp_m 0.05		
Rename			
	Cancel	OK	

 σ_{gb}



•

k_{sgb}

Hall-Petch equation



 $D\,$ - Grain diameter

 \mathfrak{H} - Subgrain diameter

 k_{p} - Constant

	Precipitation domains	? >
Precipitation domains nickelmatrix	General Mech. Props MS Evolution Solute trapping Special Thermodynamic matrix phase FCC_A1	•
	Microstructure parameters equilibrium dislocation density [m-2] 1.0e11	
	initial grain diameter [m] 20e-6 elongation factor 1	
	Burger's vector automatic manual value [m] 2.5e-10	
New Remove		
	Cancel	ОК



• Hall-Petch equation



D - Grain diameter δ - Subgrain diameter

k_n	-	Constant

variables	value	^
kinetics: pd strength TGS\$*		
TGS\$nickelmatrix	3.57771e+07	~
category: kinetics: pd strength expression: TGS\$* legal unit qualifiers: *none* -> fine grain yield strength contribution in precipitation domain		

variables	value	^]
kinetics: pd strength TSGS\$*			
TSGS\$nickelmatrix	0	¥	
category: kinetics: pd strengt expression: TSGS\$nickelmatrix legal unit qualifiers: *none* -> subgrain yield strength con	tribution in precipitation domain		



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- Contributions to yield strength, σ_{YS}
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 - Solid solution strengthening, σ_{ss}
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$$\boldsymbol{\sigma}_{YS} = f(\boldsymbol{\sigma}_{i}, \boldsymbol{\sigma}_{disl}, \boldsymbol{\sigma}_{gb}, \boldsymbol{\sigma}_{sgb}, \boldsymbol{\sigma}_{ss}, \boldsymbol{\sigma}_{prec})$$





- k_i Coefficient for element i
- C_i Element i content in the prec. Domain (mole fraction)
- n_i Exponent for element i
- m_{sub} Exponent for substitutional elements
- $m_{\rm int}$ Exponent for interstitial elements
- m_{tot} Global exponent



•••	2	
• F	Precipitation domains	$\frac{m_{ot}}{m_{sub}}$ $\frac{m_{ot}}{m_{int}}$ m_{tot}
Precipitation domains	General Mech. Props MS Evolution Solute trapping Special	$\left(\sum_{n} \left(\frac{1}{2} - n \right) m_{sub} \right)^{min} \left(\sum_{n} \left(\frac{1}{2} - n \right) m_{int} \right)^{min}$
nickelmatrix	General Solid Solution Precipitation	$\sigma_{ss} = \left \sum_{i} (k_i c_i^{n_i})^{sub} \right + \sum_{i} (k_i c_i^{n_i})^{sub} $
	Strengthening coefficients	$\left(\underbrace{-i}_{i}, \ldots, i \right)_{sub} \left(\underbrace{-i}_{i}, \ldots, i \right)_{int}$
	Element Coefficient Exponent	
	AL 225.0e6 1/2	
	C 1061.0e6 1/2 CO 39.4e6 1/2	
	CR 337e6 1/2	
	FE 153.0e6 1/2	K_i - Coefficient for element <i>i</i>
	NB 1183.0e6 1/2	l
	NI 0.0 1/2	C_{\cdot} - Element i content in the prec. domain
	TI 775.0e6 1/2 W 977.0e6 1/2	
		n_{i} - Exponent for element <i>i</i>
		r_i Exponention element r
	SSS coupling coefficients	M Experient for substitutional elements
New	substitutional (1.8) 1.8 interstitial (1.8) 1.8	m_{sub} - Exponent for substitutional elements
New Remove	total SS strength (1.8) 1.8	$\mathcal{W}_{\mathcal{W}}$
Rename		$\mu_{\rm int}$ - Exponent for interstitial elements
	Cancel OI	
		<i>m_{tot}</i> - Global exponent
Page ■ 18		





variables	value	^	
A kinetics: pd strength A TSSS\$*			
TSSS\$nickelmatrix	2.82649e+08	¥	
category: kinetics: pd strength expression: TSSS\$* legal unit qualifiers: *none*			

-> solid solution yield strength contribution in precipitation domain



• Solid solution strengthening, σ_{ss}



variables	value	^
kinetics: pd strength		
TSSS_EL\$*\$*		
TSSS_EL\$nickelmatrix\$*		
TSSS_EL\$nickelmatrix\$VA	0	
TSSS_EL\$nickelmatrix\$AL	2.78874e+07	
TSSS_EL\$nickelmatrix\$C	4.56667e+07	
TSSS_EL\$nickelmatrix\$CO	1.27085e+07	
TSSS_EL\$nickelmatrix\$CR	1.70234e+08	
TSSS_EL\$nickelmatrix\$FE	5.49895e+07	
TSSS_EL\$nickelmatrix\$MO	1.47716e+08	
TSSS_EL\$nickelmatrix\$NB	8.44694e+07	
TSSS_EL\$nickelmatrix\$NI	0	
TSSS_EL\$nickelmatrix\$TI	1.95807e+07	
TSSS_EL\$nickelmatrix\$W	5.71824e+07	~

category: kinetics: pd strength expression: TSSS_EL\$*\$W legal unit qualifiers: *none* -> solid solution yield strength contribution of element in precipitation domain



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 - Precipitation strenghtening, σ_{prec}

$$\boldsymbol{\sigma}_{YS} = f(\boldsymbol{\sigma}_{i}, \boldsymbol{\sigma}_{disl}, \boldsymbol{\sigma}_{gb}, \boldsymbol{\sigma}_{sgb}, \boldsymbol{\sigma}_{ss}, \boldsymbol{\sigma}_{prec})$$



- Some general parameters/settings
 - Angle between dislocation line and Burger's vector θ (edge:screw ratio, $\theta = 0$

for pure screw, $\theta = \pi/2$ for pure edge)

Phases FCC_A1 GAMMA_PRIME GAMMA_PRIME_P0	General Constraints Precipitate Nucleation Structure S Properties Strengthening MS Evolution Setup Strengthening model Precipitation strengthening Size classes coupling coefficient 1.8	Special
	Precipitation strengthening APB energy [J/m2] 0,111 number of pair dis APB: disl. repulsion strong 2.8 APB: disl. rep. wea ✓ auto SF energy [J/m2] 0.0 dislocation character angle: current value = 45 screw ✓ use linear misfit instead of vol. linear misfit [dL/L] Modulus strengthening model Nembach ▼	sk (0-1) 1 edge 0,004
Create Remove		



- Some general parameters/settings
 - Equivalent radius, r_{eq} (describes precipitate-dislocation interference area)

$$r_{eq} = \frac{\pi}{4} r_m$$

$$r_m$$
 - Precipitate mean radius



- Some general parameters/settings
 - Mean distance between the precipitate surfaces, L_S

$$L_{S} = \sqrt{\frac{\ln 3}{2\pi \sum_{class} N_{V,class} r_{m,class}} + 4r_{ss}^{2} - 2r_{ss}}$$

$$N_{V class}$$
 - Precipitate number density within the class

 $r_{m,class}$ - Precipitate mean radius within the class

$$r_{s} = \sqrt{\frac{2}{3}} \frac{\sum_{class} N_{V,class} r_{m,class}^2}{\sum_{class} N_{V,class} r_{m,class}}$$

value			
1MA_PRIME_P0 2.02552e-08			
itates			
GAMMA_PRIME_P0			
-> mean distance between randomly distributed precipitates on a single plane			
e* In randomly distributed precipitates on a			



- Derived from au_{prec}
 - Non-shearable particles (Orowan mechanism)
 - Shearable particles (weak or strong)
 - Coherency effect
 - Modulus effect
 - Anti-phase boundary effect
 - Stacking fault effect
 - Interfacial effect



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Non-shearable particles





Non-shearable particles

$$\tau_{nsh} = \frac{JGb}{2\pi\sqrt{1-\upsilon}L_s} \ln\left(\frac{2r_{eq}}{r_i}\right)$$
$$J = \left(\frac{1-\upsilon\left(\cos\left(\frac{\pi}{2}-\theta\right)\right)^2}{\sqrt{1-\upsilon}}\right)$$

If
$$r_{eq} < 2r_i$$
, then

$$\tau_{nsh} = \frac{JGb}{2\pi\sqrt{1-\upsilon}L_s} \ln(2)$$

$$r_{eq} = \frac{\pi}{4} r_m \qquad r_i = 2b$$



Non-shearable particles

$$\tau_{nsh} = \frac{JGb}{2\pi\sqrt{1-\upsilon}L_s} \ln\left(\frac{2r_{eq}}{r_i}\right)$$

$$J = \left(\frac{1 - \upsilon \left(\cos\left(\frac{\pi}{2} - \theta\right)\right)^2}{\sqrt{1 - \upsilon}}\right)$$

$$r_{eq} = \frac{\pi}{4} r_m \qquad r_i = 2b$$

variables	value	^
kinetics: prec. strength		
A TAO_OROWAN\$*		
TAO_OROWAN\$GAMMA_PRIME_PO	0 7.74823e+08	~
category: kinetics: prec. strength	P0	

expression: TAO_OROWAN\$GAMMA_PRIME_P0 legal unit qualifiers: *none* -> Ashby-Orowan shear stress for impenetrable precipitates of individual phase



- Derived from au_{prec}
 - Non-shearable particles (Orowan mechanism)

• Shearable particles (weak or strong)

- Coherency effect
- Modulus effect
- Anti-phase boundary effect
- Stacking fault effect
- Interfacial effect



- Some general parameters/settings
 - Dislocation bending angle, ψ weak and strong particles



 $0^{\circ} \le \psi \le 120^{\circ} \rightarrow$ "strong" particles

 $120^{\circ} \le \psi \le 180^{\circ} \rightarrow$ "weak" particles



- Some general parameters/settings
 - Mean distance between the precipitate surfaces, L



Fig. 2. Free distance between two precipitates along dislocation line in a random array. (A) The precipitates are shearable and strong and (B) the precipitates are shearable and weak.

$$r_{v} = \sqrt{\frac{\ln 3}{2\pi \sum_{class} N_{V,class} r_{m,class}} + 4r_{ss}^2 - 2r_{ss}}$$

Strong particles





- Some general parameters/settings
 - Mean distance between the precipitate surfaces, L



Weak particles

 $L_{eff} = L_{S} \left| \cos\left(\frac{\psi}{2}\right) \right|^{-1/2}$

Fig. 2. Free distance between two precipitates along dislocation line in a random array. (A) The precipitates are shearable and strong and (B) the precipitates are shearable and weak.



- Some general parameters/settings
 - Mean distance between the precipitate surfaces, L

t	Precipitation domains	?	
Precipitation domains	General Mech. Props MS Evolution Solute trapping Special		
nickelmatrix	General Solid Solution Precipitation Precipitate retarding force auto value obtained from internal calculation pinned mobility M0' 0.0 Q' 0.0 Dislocation line tension oision line tension outer cut off=120° 120 inner cut off=2,0xb 1xb 	180 4xb	
New Remove	Precipitation strengthening coupling coefficients (1-2) shearing (1.8) 1.8 non-shearing (1.8) 1.8 total (1.4) 1.4		
Rehame	Cancel	OK	

Weak

particles $L_{eff} = L_S \left| \cos \left(\frac{\psi}{2} \right) \right|^{-1/2}$



- Some general parameters/settings
 - Dislocation line tension, T
 - Simple model

$$T = \frac{Gb^2}{2}$$

Advanced model

$$T_{strong} = \frac{Gb^2}{4\pi} \left(\frac{1 + \upsilon - 3\upsilon \sin^2 \theta}{1 - \upsilon} \right) \ln \left(\frac{L_s}{r_i} \right)$$

$$T_{weak} = \frac{Gb^2}{4\pi} \left(\frac{1 + \upsilon - 3\upsilon \sin^2 \theta}{1 - \upsilon} \right) \ln \left(\frac{L_{eff}}{r_i} \right)$$



- Some general parameters/settings
 - Dislocation line tension, T
 - Simple model



• Advanced model

$$T_{strong} = \frac{Gb^2}{4\pi} \left(\frac{1 + \upsilon - 3\upsilon \sin^2 \theta}{1 - \upsilon} \right) \ln \left(\frac{L_s}{r_i} \right)$$

varia	ables	value		^
kinetics: prec. strength				
	DLT_SIMPLE\$GAMMA_PRIME_P0	1.92907e-09		
4	DLT_WEAK\$GAMMA_PRIME_P0	1.6019e-09		
	DLT_STRONG\$GAMMA_PRIME_P0	1.46929e-09		¥
category: kinetics: prec. strength expression: DLT_SIMPLE\$* legal unit qualifiers: *none* -> dislocation line tension from simple description (1/2Gb^2)				




Shearable particles

- Some general parameters/: #
 - Dislocation line tension, T

• Simple Advanced •

	Precipitation domains ? ×
Precipitation domains	General Mech. Props MS Evolution Solute trapping Special
nickelmatrix	General Solid Solution Precipitation
	Precipitate retarding force
	✓ auto value obtained from internal calculation
	pinned mobility M0' 0.0 Q' 0.0
	Dislocation line tension
	simple (1/2Gb2) dislocation character
	advanced form outer cut off=120° 120 180
	inner cut off=2,0xb 1xb 4xb
	Precipitation strengthening coupling coefficients (1-2)
	shearing (1.8) 1.8
	non-shearing (1.8) 1.8
	total (1.4) 1.4
New Remove	
Rename	
	Cancel OK



Shearable particles

- Some general parameters/settings
 - Inner cut-off radius, r_i (multiplication of Burger's vector)

8 8 8 8 7 7	Precipitation domains ? ×
Precipitation domains	General Mech. Props MS Evolution Solute trapping Special
nickelmatrix	General Solid Solution Precipitation Precipitate retarding force Image: state of the s
	advanced form outer cut off=120° 120 180 inner cut off=2,0xb 1xb 4xb
	Precipitation strengthening coupling coefficients (1-2) shearing (1.8) 1.8
	non-shearing (1.8)
New Remove	total (1.4) 1.4
	Cancel OK

$$T_{strong} = \frac{Gb^2}{4\pi} \left(\frac{1 + \upsilon - 3\upsilon \sin^2 \theta}{1 - \upsilon} \right) \ln \left(\frac{L_s}{r_i} \right)$$

$$T_{weak} = \frac{Gb^2}{4\pi} \left(\frac{1 + \upsilon - 3\upsilon \sin^2 \theta}{1 - \upsilon} \right) \ln \left(\frac{L_{eff}}{r_i} \right)$$



- Derived from au_{prec}
 - Non-shearable particles (Orowan mechanism)
 - Shearable particles (weak or strong)
 - Coherency effect
 - Modulus effect
 - Anti-phase boundary effect
 - Stacking fault effect
 - Interfacial effect



Coherency effect

- Strain field due to precipitation/matrix misfit
 - Strong particles

$$T_{coh,strong} = \frac{\left(2\cos^2\theta + 2.1352\sin^2\theta\right)}{L_S} \left(\frac{T_{strong}^3 G \varepsilon r_m}{b^3}\right)^{1/4}$$

$$\varepsilon = \frac{2}{3} \Delta_{lin} = \frac{2}{9} \Delta_{vol}$$

[·] misfit

• Weak particles

$$\tau_{coh,weak} = \frac{\left(1.3416\cos^2\theta + 4.1127\sin^2\theta\right)}{L_S} \left(\frac{G^3\varepsilon^3 r_{eq}^3 b}{T_{weak}}\right)^{1/2} \qquad \qquad \Delta_{lin} - \text{Linear misfit} \\ \Delta_{vol} - \text{Volumetric misfit}$$



Coherency effect

	Phase status
Phases FCC_A1 GAMMA_PRIME	General Constraints Precipitate Nucleation Structure Special Properties Strengthening MS Evolution
GAMMA_PRIME_P0	Mechanical properties use same values as matrix Young's modulus [Pa] 202e9 Poisson's ratio 0.3
	□ auto vol. misfit (dV/V) 0,004*3 ✓ auto Burger's vector [m] 2.5e-10 Coherency rad. [m] 0.0 Breakable above rad. [m] 1.0
Create Remove	
Help	Cancel OK

 $\varepsilon = \frac{2}{3} \Delta_{lin} = \frac{2}{9} \Delta_{vol}$

 Δ_{lin} - Linear misfit





Coherency effect

t i i i i i i i i i i i i i i i i i i i	Phase status	?
Phases	General Constraints Precipitate Nucleation Structure Special	
FCC_A1 GAMMA_PRIME	Properties Strengthening MS Evolution	1
GAMMA_PRIME_P0	Setup	
	Strengthening model Precipitation strengthening	•
	Size dasses	
	coupling coefficient 1.8	
	Precipitation strengthening	
	APB energy [J/m2] 0,111 number of pair disi. 3 APB: disi repulsion strong 2.8 APB: disi rep. weak (0-1) 1	
	✓ auto SF energy [J/m2] 0.0	
	dislocation character angle: current value = 45 screw edge	2
	✓ use linear misfit instead of vol. linear misfit [dL/L] 0,00	04
	Modulus strengthening model Nembach 🔻	
Create Remove		
Help	Concel	OK
help	Cancer	OK

 $\varepsilon = \frac{2}{3} \Delta_{lin} = \frac{2}{9} \Delta_{vol}$



 Δ_{vol} - Volumetric misfit

MatCalc Engineering

Coherency effect

- Strain field due to precipitation/matrix misfit
 - Strong particles

$$\tau_{coh,strong} = \frac{\left(2\cos^2\theta + 2.1352\sin^2\theta\right)}{L_s} \left(\frac{T_{strong}^3 G \varepsilon r_m}{b^3}\right)^{1/4}$$

• Weak particles

$$\tau_{coh,weak} = \frac{\left(1.3416\cos^2\theta + 4.1127\sin^2\theta\right)}{L_s} \left(\frac{G^3\varepsilon^3 r_{eq}^3 b}{T_{weak}}\right)^{1/2}$$

variables value	^		
A kinetics: prec. strength A TAO COHER WEAK\$*			
TAO_COHER_WEAK\$GAMMA_PRIME_P0 8.9264e+07			
TAO_COHER_STRONG\$GAMMA_PRIME_P0 4.92374e+08	~		
category: kinetics: prec. strength expression: TAO_COHER_WEAK\$GAMMA_PRIME_P0 legal unit qualifiers: *none* -> coherency hardening shear stress for shearable weak precipitates of individual phase			



- Derived from au_{prec}
 - Non-shearable particles (Orowan mechanism)
 - Shearable particles (weak or strong)
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 - Anti-phase boundary effect
 - Stacking fault effect
 - Interfacial effect



- Elastic properties of precipitate and matrix differ \rightarrow dislocation energy inside and outside the precipitate differ
- 2 models
 - Nembach
 - Siems



- Elastic properties of pr energy inside and outs
- 2 models
 - NembachSiems

	Phase statu	S		i 📕
nases	General Constraints Precipita	te Nucleation	Structure Special	
FCC_A1	Properties Strengthening	IS Evolution		
AMMA_PRIME_P0	Setup Strengthening model Precipitation	on strengthening		•
	Size classes coupling coefficient 1.8			
	Precipitation strengthening			
	APB energy [J/m2]	0,111	number of pair disl.	3
	APB: disl. repulsion strong	2.8	APB: disl. rep. weak (0-1)	1
	✓ auto SF energy [J/m2]	0.0		
	dislocation character angle: o	current value = 45	screw	edge
	✓ use linear misfit instead of vo		linear misfit [dL/L]	0,004
	Modulus strengthening model	Nembach 💌		
			•	
Create Remove				
Help	L		Cancel	ОК



- Nembach model
 - Strong particles

$$\tau_{\mathrm{mod},strong} = \frac{F_{\mathrm{mod}}}{bL_{\mathrm{S}}}$$

$$F_{\text{mod}} = 0.05 \left| G - G_P \right| b^2 \left(\frac{r_{eq}}{b} \right)^{0.85}$$

• Weak particles

$$\tau_{\text{mod,weak}} = \frac{2T_{\text{weak}}}{bL_{S}} \left(\frac{F_{\text{mod}}}{2T_{\text{weak}}}\right)^{3/2}$$

 G_p - Particle shear modulus



Weak

Modulus effect

• Siems model







- Elastic properties of precipitate and matrix differ \rightarrow dislocation energy inside and outside the precipitate differ
- 2 models
 - Nembach
 - Siems

	$ au_{\mathrm{mod},\mathit{weak}}$
-	7
	$\mathcal{U}_{\mathrm{mod},strong}$





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Anti-phase boundary (APB) effect

• Dislocation passing through ordered precipitate increases the energy

by creating the APB

• Strong particles

$$\tau_{APB,strong} = \frac{0.69}{bL_s} \left(\frac{8wT_{strong}r_{eq}\gamma_{APB}}{3}\right)^{1/2}$$

• Weak particles

$$\tau_{APB,weak} = \frac{2}{sbL_{S}} \left[2T_{weak} \left(\frac{r_{eq} \gamma_{APB}}{T_{weak}} \right)^{3/2} - \frac{16\beta \gamma_{APB} r_{eq}^{2}}{3\pi L_{S}} \right]$$

 γ_{APB} - APB energy

 w, β - Interaction parameter between

the leading and trailing dislocation

 $_{S}$ - Number of pair dislocations

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Anti-phase boundary (APB) effect

• • • • • • • • • • • • • • • • • • •	Phase status	? ×	
Phases FCC_A1 GAMMA_PRIME GAMMA_PRIME_P0	General Constraints Precipitate Nucleation Structure Special Properties Strengthening MS Evolution Structure Special Setup Strengthening model Precipitation strengthening Strengthening		cipitate increases the energy
	Size classes coupling coefficient 1.8 Precipitation strengthening APB energy [J/m2] 0,111 number of pair disl.	3	
	APB: disl. repulsion strong 2.8 APB: disl. rep. weak (0-1) ✓ auto SF energy [J/m2] 0.0 dislocation character angle: current value = 45 screw ✓ use linear misfit instead of vol. linear misfit [dL/L] Modulus strengthening model Nembach) 1 edge 0,004	γ_{APB} - APB energy <i>w</i> . β - Interaction parameter between
Create Remove			the leading and trailing dislocation <i>s</i> - Number of pair dislocations
Help	Cancel	ОК	



Anti-phase boundary (APB) effect

- Dislocation passing through ordered precipitate increases the energy
 - by creating the APB
 - Strong particles

$$\tau_{APB,strong} = \frac{0.69}{bL_s} \left(\frac{8wT_{strong}r_{eq}\gamma_{APB}}{3} \right)^1$$

• Weak particles

$$\tau_{APB,weak} = \frac{2}{sbL_{S}} \left[2T_{weak} \left(\frac{r_{eq} \gamma_{APB}}{T_{weak}} \right)^{3/2} - \frac{16\beta \gamma_{APB} r_{eq}^{2}}{3\pi L_{S}} \right]$$

variables		value	^
⊿ ki	netics: prec. strength		
1	TAO_APB_WEAK\$GAMMA_PRIME_P0	2.2558e+08	
4		7 7001 4 00	
	TAO_APB_STRONG\$GAMMA_PRIME_P0	7.78014e+08	¥

category: kinetics: prec. strength

expression: TAO_APB_WEAK\$GAMMA_PRIME_P0

legal unit qualifiers: *none*

-> anti-phase boundary hardening shear stress for weak shearable precipitates of individual phase



- Derived from au_{prec}
 - Non-shearable particles (Orowan mechanism)
 - Shearable particles (weak or strong)
 - Coherency effect
 - Modulus effect
 - Anti-phase boundary effect
 - Stacking fault effect
 - Interfacial effect



 Passing dislocation creates a stacking fault – energy difference between the SF in the precipitate and matrix

$$K_{SF} = \frac{Gb_p^2 (2 - \upsilon - 2\upsilon \cos(2\theta))}{8\pi (1 - \upsilon)}$$

$$W_{eff} = \frac{2K_{SF}}{\gamma_{SFM} + \gamma_{SFP}}$$

$$F_{SF} = 2(\gamma_{SFM} - \gamma_{SFP}) \sqrt{W_{eff} r_{eq} - W_{eff}^2/4}$$

- b_p Burger's vector of particle
- $\gamma_{\rm SFP}\,$ Stacking fault energy of particle
- γ_{SFM} Stacking fault energy of matrix



• Passing dislocation creates a stacking fault – energy difference

between the SF in the precipitate and matrix

• Strong particles

$$\tau_{SF,strong} = \frac{F_{SF}}{bL_s}$$

• Weak particles

$$\tau_{SF,weak} = \frac{2T_{weak}}{bL_{S}} \left(\frac{F_{SF}}{2T_{weak}}\right)^{3/2}$$

$$K_{SF} = \frac{Gb_p^2 (2 - \upsilon - 2\upsilon \cos(2\theta))}{8\pi (1 - \upsilon)}$$

$$W_{eff} = \frac{2K_{SF}}{\gamma_{SFM} + \gamma_{SFP}}$$

$$F_{SF} = 2(\gamma_{SFM} - \gamma_{SFP}) \sqrt{W_{eff} r_{eq} - W_{eff}^2/4}$$



*** ***	Precipitation domains	? ×	- energy difference
Precipitation domains nickelmatrix New Remove Rename	General Mech. Props MS Evolution Solute trapping Special Diffusion control Energies	ri: γ γ	X b_p - Burger's vector of particle Y_{SFP} - Stacking fault energy of particle T_{SFM} - Stacking fault energy of matrix



• • • • \$**	Phase status ?	
Phases FCC_A1 GAMMA_PRIME GAMMA_PRIME_P0	General Constraints Precipitate Nucleation Structure Special Properties Strengthening MS Evolution Setup Strengthening model Precipitation strengthening Size classes coupling coefficient 1.8 Precipitation strengthening APB energy [J/m2] 0,111 number of pair disl. 3 	It – energy difference atrix
Create Remove	APB: disl. repulsion strong 2.8 APB: disl. rep. weak (0-1) 1 ✓ auto SF energy [J/m2] 0.0 0.0 edge ✓ use linear misfit instead of vol. linear misfit [dL/L] 0,004 Modulus strengthening model Nembach ▼	b_p - Burger's vector of particle γ_{SFP} - Stacking fault energy of particle γ_{SFM} - Stacking fault energy of matrix



Phase status ? ×	
Phases General Constraints Precipitate Nucleation Structure Special FCC_A1 Properties Strengthening MS Evolution MMMA_PRIME Mechanical properties use same values as matrix Young's modulus [Pa] Young's modulus [Pa] 202e9 Poisson's ratio 0.3 Structure auto vol. misfit (dV/V) auto vol. misfit (dV/V) 0.004*3 Coherency rad. [m] 0.0 Breakable above rad. [m] 1.0	It – energy difference atrix b_p - Burger's vector of particle γ_{SFP} - Stacking fault energy of particle γ_{SFM} - Stacking fault energy of matrix



• Passing dislocation creates a stacking fault – energy difference

between the SF in the precipitate and matrix

• Strong particles

$$\tau_{SF,strong} = \frac{F_{SF}}{bL_S}$$

• Weak particles





category: kinetics: prec. strength expression: TAO_SFE_WEAK\$GAMMA_PRIME_P0 legal unit qualifiers: *none* -> stacking fault energy bardening shear stress for weak shearable

-> stacking fault energy hardening shear stress for weak shearable precipitates of individual phase



- Derived from au_{prec}
 - Non-shearable particles (Orowan mechanism)
 - Shearable particles (weak or strong)
 - Coherency effect
 - Modulus effect
 - Anti-phase boundary effect
 - Stacking fault effect
 - Interfacial effect



Interfacial effect

- Passing dislocation increases the area of precipitate/matrix interface
 - Strong particles

$$\tau_{\text{int},strong} = \frac{F_{\text{int}}}{bL_{\text{S}}}$$

$$F_{\rm int} = 2\gamma_{PM} b$$

• Weak particles

$$\tau_{\text{int,weak}} = \frac{2T_{\text{weak}}}{bL_{\text{S}}} \left(\frac{F_{\text{int}}}{2T_{\text{weak}}}\right)^{3/2}$$

 γ_{PM} - Precipitate/matrix interface energy

Interfacial effect

- Passing dislocation increases
 - Strong particles

 $\tau_{\text{int},strong} = \frac{F_{\text{int}}}{bL_{\text{S}}}$

Weak particles

$$\tau_{\rm int,weak} = \frac{2T_{\rm weak}}{bL_{\rm S}} \left(\frac{F_{\rm int}}{2T_{\rm weak}}\right)^{3/2}$$

	Pha	se status			? ×
Phases FCC_A1 GAMMA_PRIME GAMMA_PRIME_P0	General Constraints Precipitate setup Phase is precipitate Parent phase: Kinetic alias name # size classes: Precipitate properties Shape factor H/D (0.1 is point of the set of the s	Precipitate Nucleation GAMMA_PRIME GAMMA_PRIME_P0 50 Edit pre blate) use interfacial energy interfacial ener	Structur	e Special bution 1.0 from planar sh yn	Initialize
Create Remove	Interface mobility [m4/Js] Driving force model for gr	owth size class based - SFF	K	1e100	
Help				Cancel	ОК

MatCalc

 γ_{PM} - Precipitate/matrix interface energy



Interfacial effect

- Passing dislocation increases the area of precipitate/matrix interface
 - Strong particles



• Weak particles



variables value Va		^
 kinetics: prec. strength TAO_CHEM_WEAK\$* TAO_CHEM_WEAK\$GAMMA_PRIME_P0 2.307 		
TAO_CHEM_WEAK\$GAMMA_PRIME_P0 2.307		
A TAO CHEM STRONGS*	19e+06	
TAO_CHEM_STRONG\$GAMMA_PRIME_P0 1.559	19e+07	~

-> chemical hardening shear stress for shearable weak precipitates of individual phase



- Evaluation
 - Identifying the regime for each precipitate separately
 - Summation of the calculated τ_{regime} of each precipitate
 - Conversion of au_{prec} to σ_{prec}



- Evaluation
 - Identifying the regime for each precipitate separately
 - Summation of the calculated $\tau_{i,regime}$ of each precipitate
 - Conversion of au_{prec} to σ_{prec}



Regime of each precipitate

• For every precipitate phase *i*: Values of τ evaluated for each of three regimes (Non-shearable, shearable weak, shearable strong)

$$\tau_{i,strong} = \left(\tau_{i,coher,strong}^{m_{sh}} + \tau_{i,\text{mod},strong}^{m_{sh}} + \tau_{i,APB,strong}^{m_{sh}} + \tau_{i,SF,strong}^{m_{sh}} + \tau_{i,\text{int},strong}^{m_{sh}}\right)^{1/m_s}$$

$$\tau_{i,weak} = \left(\tau_{i,coher,weak}^{m_{sh}} + \tau_{i,\text{mod},weak}^{m_{sh}} + \tau_{i,APB,weak}^{m_{sh}} + \tau_{i,SF,weak}^{m_{sh}} + \tau_{i,\text{int},weak}^{m_{sh}}\right)^{1/m_{sh}}$$

 $\tau_{i,nsh}$



?

Regime of each precipitate

- For every precipitate ph
 - regimes (Non-shearable

$$\tau_{i,strong} = \left(\tau_{i,coher,strong}^{m_{sh}} + \tau_{i,\text{mod}}^{m_{sh}}\right)$$
$$\tau_{i,weak} = \left(\tau_{i,coher,weak}^{m_{sh}} + \tau_{i,\text{mod},n}^{m_{sh}}\right)$$

eimatrix	General Solid Solution Precipitation	
	Precipitate retarding force	
	✓ auto value obtained from internal calculation	
	pinned mobility M0' 0.0 Q' 0.0	
	Dislocation line tension	
	simple (1/2Gb2) dislocation character	
	advanced form outer cut off=120° 120	
	inner cut off=2,0xb 1xb 4xb	b
	Precipitation strengthening coupling coefficients (1-2)	
	shearing (1.8) 1.8	
	non-shearing (1.8) 1.8	
	total (1.4) 1.4	
w Remove		

Dracinitation domains



Regime of each precipitate

• For every precipitate phase *i*: Values of τ evaluated for each of three

regimes (Non-shearable, shearable weak, shearable strong)

$$\tau_{i,strong} = \left(\tau_{i,coher,strong}^{m_{sh}} + \tau_{i,MDd,strong}^{m_{sh}} + \tau_{i,SF,strong}^{m_{sh}} + \tau_{i,int,strong}^{m_{sh}}\right)^{1/m_{sh}}$$

$$\tau_{i,weak} = \left(\tau_{i,coher,weak}^{m_{sh}} + \tau_{i,MDd,weak}^{m_{sh}} + \tau_{i,SF,weak}^{m_{sh}} + \tau_{i,int,weak}^{m_{sh}}\right)^{1/m_{sh}}$$

$$\tau_{i,nsh}$$





- Evaluation
 - Identifying the regime for each precipitate separately
 - Summation of the calculated $\tau_{i,regime}$ of each precipitate
 - Conversion of au_{prec} to σ_{prec}


Summation of $\tau_{i,regime}$



 $\tau_{i,sh}$ -Shearable particles contribution (weak and strong regime)

 $\tau_{i,nsh}$ -Non-shearable particles contribution



Summation of $\tau_{i,regime}$

•



cipitation domains	General Mech. Props MS Evolution Solute trapping Special
ckelmatrix	General Solid Solution Precipitation
	Precipitate retarding force
	✓ auto value obtained from internal calculation
	pinned mobility M0' 0.0 Q' 0.0
	Dislocation line tension
	Simple (1/2Gb2) dislocation character
	advanced form outer cut off=120° 120
	inner cut off=2,0xb 1xb 4xb
	Precipitation strengthening coupling coefficients (1-2)
	shearing (1.8) 1.8
	non-shearing (1.8) 1.8
	total (1.4) 1.4
New Remove	
Rename	
	Cancel OK



Summation of $\tau_{i,regime}$

- Precipitation strengthening, σ_{prec} (derived from τ_{prec})
 - Evaluation of au_{prec}



variables		value	^
⊿ kir	netics: pd strength		
4	TAO NON SHEAR\$*		
	TTAO_NON_SHEAR\$nickelmatrix	4.47201e+08	
4	TTAO_SHEAR\$*		
	TTAO_SHEAR\$nickelmatrix	1.81568e+08	
4	TAO PRECS*		
	TTAO_PREC\$nickelmatrix	5.34359e+08	~

category: kinetics: pd strength expression: TTAO_NON_SHEAR\$nickelmatrix legal unit qualifiers: *none* -> total shear stress from non-shearable precipitates



Precipitation strengthening, σ_{prec}

- Evaluation
 - Identifying the regime for each precipitate separately
 - Summation of the calculated $\tau_{i,regime}$ of each precipitate

• Conversion of au_{prec} to σ_{prec}



Precipitation strengthening, σ_{prec}

• Evaluation

Page **77**

- Identifying the regime for each precipitate separately
- Summation of the calculated $\tau_{i,regime}$ of each precipitate
- Conversion of au_{prec} to σ_{prec}

$$\sigma_{prec} = lpha au_{prec}$$

$$\chi$$
 - Taylor factor

variables	value	^
A kinetics: pd strength A TSIGMA PRECS*		
TSIGMA_PREC\$nickelmatrix	1.23579e+09	~
category: kinetics: pd strength expression: TSIGMA_PREC\$nickelmatrix legal unit qualifiers: *none* -> total vield strength contribution from p	precipitates	



Total yield strength, σ_{YS}

• Summation of contributions

$$\sigma_{YS} = \left[\sigma_{disl}^{m_{sig}} + \left(\sigma_{i} + \sigma_{gb} + \sigma_{sgb} + \sigma_{ss} + \sigma_{prec}\right)^{m_{sig}}\right]^{1/m_{sig}}$$



Total yield strength, σ_{YS}

Precipitation

New ...

Summation of contributions #

$\sigma_{\scriptscriptstyle YS}$ =	$= \sigma_{disl}^{m_{sig}}$	$+(\sigma_i +$	$\sigma_{_{gi}}$

	General Solid Solution Precipitation
	Mechanical properties
	Young's Modulus [Pa] 208e9
	Taylor factor (2.5-3.1) 2,6 Poisson's ratio ,33
	Matrix strength evaluation
	Basic strength [Pa] 21,8e6
	Hall-Petch coeff (gb/sgb) 0,16e6 / 0.0e6
	Disl. strengt. coeff. (a1/a2) 0.5 / 0.3
	Total strength coupling coefficients
	Coeff. thermal + athermal (1.8) 1.8
	Strain rate sensitivity (temporary)
	eps_dot_ref 1.0 exp_m 0.05
Remove	
Rename	



Total yield strength, σ_{YS}

• Summation of contributions

$$\sigma_{YS} = \left[\sigma_{disl}^{m_{sig}} + \left(\sigma_{i} + \sigma_{gb} + \sigma_{sgb} + \sigma_{ss} + \sigma_{prec}\right)^{m_{sig}}\right]^{1/m_{sig}}$$

variables value		^	
⊿ kin ⊿]	etics: pd strength TVS\$*		
	TYS\$nickelmatrix	1.58386e+09	~
categ expre legal -> to	category: kinetics: pd strength expression: TYS\$nickelmatrix legal unit qualifiers: *none* -> total yield strength of precipitation domain		



Thank you for your attention !





2

×

Precipitation hardening

Shape factor influence



Phase status ...



Shape factor influence on L_S

$$L_{s} = K \left(\sqrt{\frac{\ln 3}{2\pi \sum_{class} N_{V,class} r_{m,class}} + 4r_{ss}^{2}} - 2r_{ss} \right)$$
$$K = h^{1/6} \left(\frac{2 + h^{2}}{3} \right)^{-1/4}$$

h - Shape factor



Figure 2. Variation in particle distances and strengthening for prolate and oblate precipitates relative to spherical particles.

Sonderegger B., Kozeschnik E., Scripta Mater., 66 (2012) 52-55

MatCalc Engineering

Precipitation hardening

Shape factor influence on L_S

$$L_{s} = K \left(\sqrt{\frac{\ln 3}{2\pi \sum_{class} N_{V,class} r_{m,class}} + 4r_{ss}^{2}} - 2r_{ss} \right)$$

$$\left(2 + h^{2} \right)^{-1/4}$$

variables	value	^	
 kinetics: precipitates L_MEAN_2D\$* L_MEAN_2D\$GAMMA_PRIME_P0 	2.02552e-08	~	
ategory: kinetics: precipitates expression: L_MEAN_2D\$GAMMA_PRIME_P()		

legal unit qualifiers: *none* -> mean distance between randomly distributed precipitates on a single plane (2-dimensional)

h - Shape factor

 $K = h^{1/6} \left(\frac{2+n}{3} \right)$

Sonderegger B., Kozeschnik E., Scripta Mater., 66 (2012) 52-55



• Non-shearable particles (Orowan mechanism)

$$\tau_{Orowan} = \frac{JGb}{2\pi\sqrt{1-\upsilon}L_s} \ln\left(\frac{2r_{eq}}{r_i}\right) \qquad r_{eq} = \left(P_{edge}r_{eq,edge,ns} + P_{screw}r_{eq,screw,r}\right)$$

$$r_{eq,edge,ns} = \left[\frac{2h^{2/3}}{3}\left(\sqrt{\frac{3}{2+h^2}} + \sqrt{\frac{3}{h^2}} + \frac{3}{2+h^2}}\right)\right]\frac{\pi}{4}r_m$$

 $r_{eq,screw,ns} = \left[\frac{2h^{2/3}}{3}\left(\frac{1}{h} + \sqrt{\frac{1}{h^2} + \frac{9}{2+h^2}}\right)\right] \frac{\pi}{4}r_m$

$$e_{q,edge,ns}$$
 - Equivalent radius for edge disl.

- $r_{eq,screw,ns}$ Equivalent radius for screw disl.
 - P_{edge} Fraction of edge disl.



Shearable particles – (e.g. coherency effect)

$$\tau_{coh,weak} = \frac{f(\theta)}{L_s} \left(\frac{G^3 \varepsilon r_{eq}^3 b}{27T_{weak}}\right)^{1/2} h$$

$$r_{eq,edge,sh} = \left[\frac{h^{2/3}}{3} \left(\sqrt{\frac{3}{2+h^2}} + 2\sqrt{\frac{6}{1+5h^2}}\right)\right] \frac{\pi}{4} r_m$$

$$r_{eq,screw,sh} = \left[\frac{h^{2/3}}{3}\left(\frac{1}{h} + 2\sqrt{\frac{2}{1+h^2}}\right)\right]\frac{\pi}{4}r_m$$

$$r_{eq} = \left(P_{edge}r_{eq,edge,sh} + P_{screw}r_{eq,screw,sh}\right)$$

 $r_{eq,edge,sh}$ - Equivalent radius for edge disl.

 $r_{eq,screw,sh}$ - Equivalent radius for screw disl.



- Shearable particles Coherency effect for non-spherical particles
 - Strong particles

$$\tau_{coh,strong} = \underbrace{(1.1101\cos^2\theta + 2.1488\sin^2\theta)}_{L_S} \left(\frac{T_{strong}^3 G \varepsilon r_m}{b^3}\right)^{1/4} K \qquad K = h^{1/6} \left(\frac{2+h^2}{3}\right)^{-1/4}$$

• Weak particles

$$\tau_{coh,weak} = \underbrace{(2.7310\cos^2\theta + 3.4736\sin^2\theta)}_{L_s} \left(\underbrace{\frac{G^3 \varepsilon r_{eq}^3 \phi}{27 r_{weak}}}_{27 r_{weak}} \right)^{1/2} h \quad \text{,if} \quad h \le \frac{(1.3416\cos^2\theta + 4.1127\sin^2\theta)}{(2.7310\cos^2\theta + 3.4736\sin^2\theta)}$$



 $\mathbf{\mathcal{N}}$

Summing up...

- Total yield strength, σ_{YS}
 - Summation of contributions

$$\sigma_{YS} = \left[\sigma_{disl}^{m_{sig}} + \left(\sigma_{i} + \sigma_{gb} + \sigma_{sgb} + \sigma_{ss} + \sigma_{prec}\right)^{m_{sig}}\right]^{1/m_{sig}}$$

• Influence of strain rate, \dot{arphi}

$$\sigma_{YS} = \left[\sigma_{disl}^{m_{sig}} + \left(\sigma_{i} + \sigma_{gb} + \sigma_{sgb} + \sigma_{ss} + \sigma_{prec}\right)^{m_{sig}}\right]^{1/m_{sig}} \left(1 + \frac{\dot{\varphi}}{C_{1}}\right)^{C_{2}}$$



Summing up...

**	Precipitation domains ?	×
Tot Precipitation domains	General Mech. Props MS Evolution Solute trapping Special	
nickelmatrix	General Solid Solution Precipitation	
•	Mechanical properties	
	Young's Modulus [Pa] 208e9	-
	Taylor factor (2.5-3.1) 2,6 Poisson's ratio ,33	$1/m_{sig}$
C	Matrix strength evaluation	ng
	Basic strength [Pa] 21,8e6	-
	Hall-Petch coeff (gb/sgb) 0,16e6 / 0.0e6	
•	Disl. strengt. coeff. (a1/a2) 0.5 / 0.3	
	Total strength coupling coefficients	
	Coeff. thermal + athermal (1.8) 1.8	
	Strain rate sensitivity (temporary)	$m \cdot (\dot{\mathbf{n}})^{\mathbf{c}_2}$
<u> </u>	eps_dot_ref 1.0 exp_m 0.05	$1 + \frac{\varphi}{\varphi}$
YS New Remo	ve	
Renam	· · · · · · · · · · · · · · · · · · ·	(U_1)
	Cancel OK	
age = 90		.:



Summing up...

- Total yield strength, σ_{YS}
 - Summation of contributions

$$\sigma_{YS} = \left[\sigma_{disl}^{m_{sig}} + \left(\sigma_{i} + \sigma_{gb} + \sigma_{sgb} + \sigma_{ss} + \sigma_{prec}\right)^{m_{sig}}\right]^{1/m_{sig}}$$

• Influence of strain rate, \dot{arphi}

legal unit qualifiers: *none* -> total yield strength of precipitation domain

$$\sigma_{YS} = \left[\sigma_{disl}^{m_{sig}} + \left(\sigma_{i} + \sigma_{gb} + \sigma_{sgb} + \sigma_{ss} + \sigma_{prec}\right)^{m_{sig}}\right]^{1/m_{sig}} \left(1 + \frac{\dot{\varphi}}{C_{1}}\right)^{C_{2}}$$